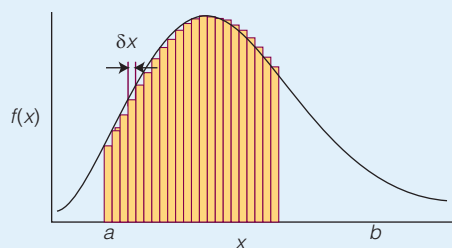


The chemist's toolkit 2 Integration

Integration is the process of adding together an infinite number of infinitesimal quantities to calculate their total contribution to a property or process (the contributions are 'integrated' into a total). It is denoted by the symbol \int , which is just an elongated S to denote a special kind of sum. Specifically, the **integral** of a function $f(x)$, between the two locations $x = a$ and $x = b$ is defined by imagining the x -axis between $x = a$ and $x = b$ as divided into strips of infinitesimal width dx , evaluating the function at the edge of each strip, calculating the area, $f(x) \times dx$, of each rectangular strip, and then adding together all those infinitesimal areas. As can be appreciated from the sketch, the integral is the area under the curve between the limits a and b . The integral so evaluated is denoted

$$\int_a^b f(x) dx$$



and is called the **definite integral** of $f(x)$ ('definite', because the beginning and end of the range of integration have been defined). The function $f(x)$ itself is called the **integrand**.

In many cases, $f(x)$ has a definite algebraic form, such as $f(x) = kx$, and its integrated form is well known and listed in tables, like those in the *Resource section*. In many of these cases, the limits of integration (a and b in the above expression) can be left undefined, and the resulting expression is called the **indefinite integral** of $f(x)$. For instance, if $f(x) = kx$, then its² indefinite integral is $kx^2 + C$, where C is a constant. If the indefinite integral of $f(x)$ in general is $g(x) + C$, then the corresponding definite integral is evaluated by writing

$$\begin{aligned} \int_a^b f(x) dx &= \{g(x) + C\} \Big|_a^b = \{g(b) + C\} - \{g(a) + C\} \\ &= g(b) - g(a) \end{aligned}$$

Note that the constant of integration disappears.

The definite and indefinite integrals encountered in this text are listed in the *Resource section*. They may also be calculated by using mathematical software. When the integrand does not have one of the listed forms, it can be evaluated numerically by using procedures included in all mathematical software packages.